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压电式振动发电机的建模及应用

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摘要：振动式压电发电机是一种可为旋转机械故障诊断中的无线传感节点供电的微能源。本文设计了由压电悬臂梁、质量块和固定装置组成的振动式压电发电机，并建立了安装在恒速旋转机械上的悬臂梁式发电机的数学模型。分析了轴向分力对悬臂梁刚度和发电机频率的影响，在考虑悬臂梁受到轴向力影响的基础上，给出了发电机固有频率、输出电压和输出功率的公式。对一个安装于转动框架上的振动式压电发电机进行了测量，结果表明，当框架转动频率为 14.25 Hz 时，发电机的输出功率约为 35 μW，随着旋转机械转动频率偏离发电机固有频率，发电机的输出功率很快降低。

关　键　词：能量获取；振动发电机；压电发电机；微机电系统
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Modeling and application of piezoelectric vibration-based power generator

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Abstract： The piezoelectric vibration-based power generator is a promising MEMS electricity source to power wireless sensing nodes for the fault diagnosis of rotary machines. In this paper，a piezoelectric vibration-based power generator is designed and the mathematical model of the cantilever piezoelectric generator used in rotary mechines is established. Then，the effects of the axial extension force on the stiffness of the beam and the frequency of the generator are analyzed，and the formulas of the frequency，output voltage and output power of the cantilever piezoelectric generator are derived. The output property of the piezoelectric vibration-based generator fixed on a rotary frame is measured. Obtained results show that the maximum output power of the generator is about 35 μW at the rotation frequency about 14.25 Hz. The output power decreases quickly when the rotation frequency deviates from the frequency of the generator.

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Key words: energy harvesting; vibration generator; piezoelectric generator; MEMS

1 Introduction

Nowadays, most wireless sensing nodes are powered by batteries, which need to be replaced or recharged because of their short lifetimes. For wireless sensing networks (WSN) with thousands of sensing nodes located in the remote or dangerous regions, it is tedious or impossible to replace or recharge these batteries. The power has become the bottleneck technology for the application of WSN. The MEMS electricity sources based on the harvest of the external environment energy have attracted more attentions for the advantages of low costs, long lives, maintenance-free and so on. The vibration-based microgenerator which can convert the environmental vibration energy into electricity power is one of the most promising MEMS electricity devices for WSN[1-2]. So far, three types of vibration-based generators, electromagnetic, electrostatic and piezoelectric ones have been reported in literatures[1-8]. The piezoelectric generator receives more and more attentions in recent years for its advantages, such as simple structure, high energy density, easy to be machined and so on. Roundy et al designed a piezoelectric vibration-based generator with the volume of 1 cm^3 which generated 375 μW electrical power from a vibration source of 2.5 $\mathrm{m/s}^2$ at 120 Hz[4].

For rotary machines such as a huge electromotor, the vibration information of the rotary parts can be used to monitor the real-time status of the machines[9]. The online fault diagnosis technology based-on vibration measurement is of importance to discover these faults in advance to avoid huge losses of lives and assets. The wireless sensing technology has to be used when the sensing nodes are installed on the rotary parts such as the vanes of a huge electromotor. The vibration-based generator has a potential to power these wireless sensing nodes because it produces electricity power when the machine rotates in the gravity field. The mathematical model of cantilever piezoelectric generator has been established[2, 4, 6], but only a few literatures reported its mathematical model when it is used in rotary machines. The mathematical model of the cantilever piezoelectric generator used in rotary machines is established at first, and then the experimental results are presented.

2 Mathematical model of cantilever piezoelectric generator

2.1 Structure and operational mechanism

A vibration-based cantilever piezoelectric generator is schematically showed in Fig. 1. The generator is composed of a cantilever, a fixture and a proof mass. The upper layer and the bottom layer of the cantilever are composed of piezoelectric materials. The middle layer is a metal layer as the electrode between the upper and the bottom piezoelectric layers. The proof mass is at the free end the cantilever and the other end of the cantilever is fixed by the fixture. With the rotation of the machine, the components, perpendicular to the beam, of the gravity force and of the inertial force of the proof mass cause the beam to deform and vibrate. This deformation causes the change of the stress in the piezoelectric layers. The change of the stress causes the change of the voltage between the two electrodes of the piezoelectric layer because of the piezoelectricity effects. This voltage can power a load. When the frequency of the rotation is close to the frequency of the generator, the output power is large.

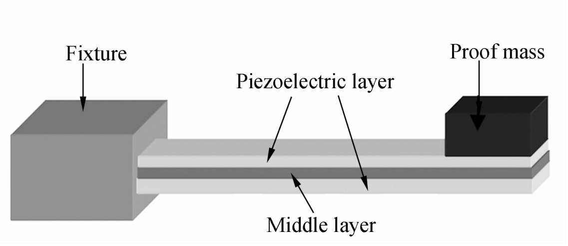

Fig. 1 Schematic of cantilever piezoelectric generator

2.2 Mathematical model of a cantilever piezoelectric generator used in rotary machines

Roundy et al[4] established the model of the piezoelectric generator by using the deflection at the joint point of the cantilever and the mass. Because the difference between the deflections at the joint point and the center of the mass is neglected, Roundy model is inaccurate when the length of the mass is larger than that of the cantilever. To extend the applicable range, He et al[6] established two models of the cantilever piezoelectric generator by simplifying the generator as a single-degree-of-freedom system and two-degree-of- freedom system respectively, which the difference between the deflections at the center of the proof mass and at the end of the cantilever has been considered.

When the cantilever piezoelectric generator is fixed at the clamped point on the rotation part of a rotary machine rotating at a constant angular velocity of ω_0, it is assumed that the neutral axis of the beam coincides with the radial direction of the rotation. The centrifugal force of the mass and the component in parallel with the cantilever of the gravity force of the proof mass exert an axial extension force onto the beam. The components, perpendicular to the cantilever, of the inertial force and the gravity force of the proof mass cause the beam to vibrate. The model is schematically shown in Fig. 2. The axial direction is denoted by x. Let $y(x)$ denote the deflection of cantilever along its neutral axis, l_b and l_m respectively the length of the cantilever and the mass, t_C and t_{sh} respectively the thickness of the piezoelectric layers and that of the middle metal layer, w and m respectively the width of the cantilever and the mass of the proof mass, and r_0 the distance between the rotation center and the clamped point. It is assumed that only the part with the length of l_e near the clamped point of the cantilever is covered with metal layers, and only this part of the piezoelectric layers produces electricity.

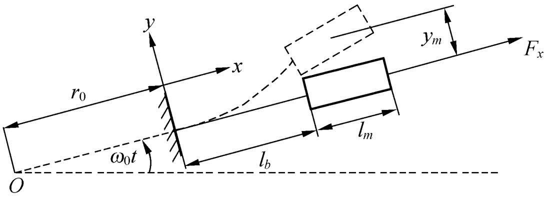

Fig. 2 Schematic of mathematical model of cantilever piezoelectric generator

Because of stress stiffness effect, the axial extension force changes the stiffness of the cantilever and the frequency of the cantilever generator. In most cases, the component perpendicular to the cantilever of the gravity force of the proof mass is much smaller than the centrifugal force of the proof mass. Compared with the influence of the centrifugal force of the mass on the stiffness of the cantilever, that of the component perpendicular to the cantilever of the inertial force of the mass can be neglected. The axial extension force is approximately given by

$$F_x \approx m\omega_0^2. \tag{1}$$

In the following, the force produced by the flow of the gas can be neglected. Let P and M respectively denote the shear force and the bending moment on the cantilever at its joint point with the proof mass. The total moment on the beam is given by

$$M_t(x) = P(l_b - x) + M - F_x y(x) \ , \tag{2}$$

where the third term in the right side of the equation denotes the moment produced by the axial extension force. When the deflection of the beam is small, the differential equation of the beam is

$$y''+\frac{F_x}{EI}y=\frac{Pl_b+M}{EI}-\frac{Px}{EI}\ , \qquad (3)$$

where E and I respectively denote the elastic modulus and the sectional inertial moment of the beam. The solution of Eq. (3) with the boundary conditions of $y|_{x=0}=y'|_{x=0}=0$ is given by

$$y=-\frac{Pl_b+M}{F_x}\cos(\sqrt{\frac{F_x}{EI}}x)+\frac{P}{F_x}\sqrt{\frac{EI}{F_x}}\sin(\sqrt{\frac{F_x}{EI}}x)-\frac{P}{F_x}x+\frac{Pl_b+M}{F_x}\ . \qquad (4)$$

When F_x approaches 0, the above solution approaches the deflection of a cantilever without the axial force. The deflection and the rotation angle at the end of the cantilever are given by

$$y_1=\frac{P}{F_x}\left[\sqrt{\frac{EI}{F_x}}\sin(\sqrt{\frac{F_x}{EI}}l_b)-l_b\cos(\sqrt{\frac{F_x}{EI}}l_b)\right]+\frac{M}{F_x}\left[1-\cos(\sqrt{\frac{F_x}{EI}}l_b)\right]\ , \qquad (5)$$

$$\alpha_1=\frac{P}{F_x}\left[l_b\sqrt{\frac{F_x}{EI}}\sin(\sqrt{\frac{F_x}{EI}}l_b)+\cos(\sqrt{\frac{F_x}{EI}}l_b)-1\right]+\frac{M}{F_x}\sqrt{\frac{F_x}{EI}}\sin(\sqrt{\frac{F_x}{EI}}l_b)\ . \qquad (6)$$

When P and M are exerted by the components of the gravity and the inertial force of the mass, we can get

$$P=-mg\sin\omega_0 t-m\ddot{y}\ , \qquad (7)$$

$$M=-0.5l_m mg\sin\omega_0 t-0.5l_m m\ddot{y}\ . \qquad (8)$$

Substituting Eqs. (4), (7) and (8) into Eq. (2), we get the total bend moment $M_t(x)$. When the average stress in the upper piezoelectric layer is denoted by $\bar{\sigma}=(mg\sin\omega_0 t+m\ddot{y})/b^{**}$[4], we obtain

$$b^{**}=\frac{1}{(l_b+0.5l_m)\frac{1}{l_e}\sqrt{\frac{EI}{F_x}}\sin\sqrt{\frac{F_x}{EI}}l_e-\frac{1}{l_e}\frac{EI}{F_x}(\cos\sqrt{\frac{F_x}{EI}}l_e-1)}. \qquad (9)$$

When the deflection at the center of the mass is denoted by $y_m=-\bar{\varepsilon}/b^*$[4], where is the average strain in the upper piezoelectric layer. We get

$$b^*=\frac{(l_b+0.5l_m)\sqrt{EIF_x}\sin\sqrt{\frac{F_x}{EI}}l_e-EI(\cos\sqrt{\frac{F_x}{EI}}l_e-1)}{El_e\left[(0.25l_m^2\sqrt{\frac{F_x}{EI}}+0.5l_m l\sqrt{\frac{F_x}{EI}}+\sqrt{\frac{EI}{F_x}})\sin(\sqrt{\frac{F_x}{EI}}l)-l\cos(\sqrt{\frac{F_x}{EI}}l)\right]}\ . \qquad (10)$$

For the proof mass, the stiffness of the supporting beam is $K_m=Eb^*b^{**}$ and the frequency of the cantilever generator is given by

$$f=\frac{1}{2\pi}\sqrt{\frac{Eb^*b^{**}}{m}}\ . \qquad (11)$$

By establishing the equivalent circuit model of the piezoelectric generator[4], such parameters as the output voltage and the output power can be derived. When the generator rotates with the frequency of ω_0, the output voltage on a resistor with the resistance of R is given by

$$U=\frac{-\mathrm{j}\omega\,\frac{2Y_C d_{31} t_c b^{*} b^{**}}{a\varepsilon b_1^{**}}g}{\left[-\frac{Y_C b^{*} b^{**}}{mRC_p}-\left(\frac{1}{RC_p}+4\pi\xi f\right)\omega_0^2\right]+\mathrm{j}\omega_0\left[\frac{Y_C b^{*} b^{**}}{m}(1+k^2)+\frac{4\pi\xi f}{RC_p}-\omega_0^2\right]}, \tag{12}$$

where ε and d_{31} are respectively the permittivity and piezoelectric strain constant of the piezoelectric material, C_p is the capacitance of the generator, $k_{31}=d_{31}\sqrt{Y_C/\varepsilon}$ is the couple coefficient, and g is the gravity accelerometer. When the upper and the bottom piezoelectric layer is in series, $a=1$. When they are in parallel, $a=2$. When $\omega_0=2\pi f$, the output voltage is given by

$$U=\left(-\mathrm{j}\,\frac{2Y_C d t_c b^{*} g}{a\varepsilon}\right)\Bigg/\left(-2\xi\omega_0^2+\mathrm{j}\left(\frac{2\omega_0\xi}{RC_p}-\omega_0^2 k_{31}^2\right)\right). \tag{13}$$

The output power is

$$P=|U|^2/2R\,. \tag{14}$$

3 Experimental results

A testing system including an electromagnetic electromotor, a rotary frame, an optical counter and an oscillometer is established. Both the cantilever piezoelectric generator and the measuring circuit are fixed on the rotary frame. The optical counter and the oscillometer are used to record the angular velocity of the frame. To measure the output power of the generator, the alternating current produced by the generator is changed into direct current by a full-wave rectified circuit and a capacitor with the capacitance of 2 200 μF is charged by the direct current. By changing the voltage on the electromagnetic electromotor, the angular velocity of the frame can be controlled. For different input voltages on the electromagnetic electromotor, the voltages on the capacitor are measured after the frame has been rotating for 10 minutes. Fig. 3 is the measured output power of the generator versus the rotation frequency. According to Fig. 3, the maximum output power is about 35 μW at the rotation frequency about 14. 25 Hz. We may conclude that the frequency of the generator is about 14. 25 Hz. The output power decreases quickly when the rotation frequency deviates from 14. 25 Hz.

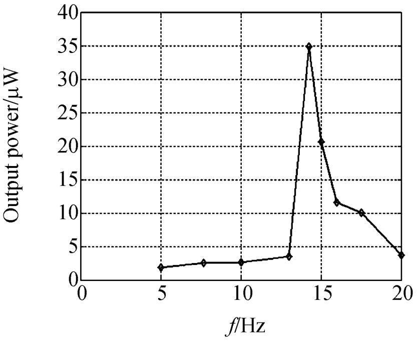

Fig. 3 Measured output power *vs* rotation speed

4 Conclusions

The piezoelectric vibration-based power generator fixed on a rotary machine produces electricity energy when the machine rotates in gravity field. It is a very promising electricity source to power wireless sensing nodes for the fault diagnosis of rotary machines. For a cantilever piezoelectric generator fixed on a rotary machine rotating with a constant angular velocity, the effects of the axial extension force on the stiffness of the beam and on the frequency of the generator are analyzed. The formulas of the frequency, the output voltage and the output power of the cantilever piezoelectric generator are generated. For a cantilever piezoelectric generator fixed on a rotary frame, the measured maximum output power of the generator is about 35 μW at the rotation frequency about 14. 25 Hz. Experimental results show that the output power decreases quickly when the rotation frequency deviates from the frequency of the generator.

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